New Superstring Isometries and Hidden Dimensions

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Abstract

We study the hierarchy of hidden space-time symmetries of noncritical strings in RNS formalism, realized nonlinearly. Under these symmetry transformations the variation of the matter part of the RNS action is cancelled by that of the ghost part. These symmetries, referred to as the α-symmetries, are induced by special space-time generators, violating the equivalence of ghost pictures. We classify the α-symmetry generators in terms of superconformal ghost cohomologies $H_n \sim H_{-n-2}(n \geq 0)$ and associate these generators with a chain of hidden space-time dimensions, with each ghost cohomology $H_n \sim H_{-n-2}$ "contributing" an extra dimension. Namely, we show that each ghost cohomology $H_n \sim H_{-n-2}$ of non-critical superstring theory in d-dimensions contains $d+n+1$ α-symmetry generators and the generators from $H_k \sim H_{-k-2}, 1 \leq k \leq n$, combined together, extend the space-time isometry group from the naive $SO(d,2)$ to $SO(d+n,2)$. In the simplest case of $n=1$ the α-generators are identified with the extra symmetries of the $2T$-physics formalism, also known to originate from a hidden space-time dimension. **PACS:**$04.50.+h$;$11.25.Mj$.

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1. Introduction

In our recent work [1] we have shown that non-critical RNS superstring theories are invariant under the set of unusual nonlinear space-time transformations, not at all evident from the structure of their worldsheet actions. That is, consider the worldsheet action of d-dimensional non-critical RNS superstring theory given by

$$\begin{aligned} S = \frac{1}{2\pi}\int d^2z\{-\frac{1}{2}\partial X_m\bar{\partial}X^m - \frac{1}{2}\psi_m\bar{\partial}\psi^m - \frac{1}{2}\bar{\psi}_m\partial\bar{\psi}^m\} + S_{ghost} + S_{Liouville} \\ S_{ghost} = \frac{1}{2\pi}\int d^2z\{b\bar{\partial}c + \bar{b}\partial\bar{c} + \beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma}\} \\ S_{Liouville} = \frac{1}{4\pi}\int d^2z\{\partial\varphi\bar{\partial}\varphi + \lambda\bar{\partial}\lambda + \bar{\lambda}\partial\bar{\lambda} - F^2 + 2\mu_0 be^{b\varphi}(ib\lambda\bar{\lambda} - F)\} \end{aligned} \tag{1}$$

where φ, λ, F are the components of the Liouville superfield, $X^m(m = 0, ..., d-1)$ are the space-time coordinates,$\psi^m, \bar{\psi}^m$ are their worldsheet superpartners; b, c, β, γ are the fermionic and bosonic (super)reparametrization ghosts bosonized as

$$\begin{aligned} b = e^{-\sigma}; c = e^{\sigma}, \\ \beta = e^{\chi-\phi}\partial\chi \equiv \partial\xi e^{-\phi}; \gamma = e^{\phi-\chi}. \end{aligned} \tag{2}$$

.

This action is obviously invariant under two global d-dimensional space time symmetries - Lorenz rotations and translations. One can straightforwardly check, however, that in addition to these obvious symmetries the action (1) is also invariant under the following nonlinear global transformations, mixing the matter and the ghost sectors of the theory [1]:

$$\begin{aligned} \delta X^m = \epsilon\{\partial(e^{\phi}\psi^m) + 2e^{\phi}\partial\psi^m\} \\ \delta\psi^m = \epsilon\{-e^{\phi}\partial^2 X^m - 2\partial(e^{\phi}\partial X^m)\} \\ \delta\gamma = \epsilon e^{2\phi-\chi}(\psi_m\partial^2 X^m - 2\partial\psi_m\partial X^m) \\ \delta\beta = \delta b = \delta c = 0 \end{aligned} \tag{3}$$

The variations of the matter and the ghost parts of the RNS action (1) under the transformations (3) are given by

$$\begin{aligned} \delta_{S_{matter}} = -\frac{\epsilon}{2\pi}\int d^2z(\bar{\partial}e^{\phi})\{\partial^2 X_m\psi^m - 2\partial X_m\partial\psi^m\} \\ \delta S_{ghost} \equiv \delta S_{\beta\gamma} = -\delta S_{matter} \end{aligned} \tag{4}$$

so the action is symmetric under (3), as the variation of the matter part is cancelled by that of the ghost part.

It is easy to check that the generator of the transformations (3) is given by

$$T = \int \frac{dz}{2i\pi} e^{\phi}(\partial^2 X_m \psi^m - 2\partial X_m \partial\psi^m) \tag{5}$$

The integrand of (5) is a primary field of dimension 1, i.e. a physical generator. While it is not manifestly BRST-invariant (it doesn't commute with the supercurrent terms of Q_{brst}), its BRST invariance can be restored by adding the appropriate b-c ghost dependent terms, according to the prescription described in [1]. The peculiar property of this generator is that it is annihilated by $\Gamma^{-1} = c\partial\xi e^{-2\phi}$ and has no analogues at higher pictures, such as $0, -1$ and -2 (but has versions at higher positive pictures $+2, +3, ...$ which can be obtained by using the direct picture-changing operator $\Gamma = \{Q_{brst}, \xi\}$). The physical operators with such a property are referred to as the positive ghost number $+1$ cohomology H_1, or simply ghost cohomology 1 [1].

There is however, a picture -3 version of this generator, with the manifest BRST invariance. This version can be obtained simply by replacing $e^{\phi} \to e^{-3\phi}$ in (5). Similarly to the picture $+1$-version, the picture -3 version is annihilated by Γ, so there are no versions of this operator at pictures $-2, -1$ and 0 while the versions at pictures below -3 can be obtained by straightforward inverse picture changing. For this reason, the picture -3 version of (5) is an element of negative ghost cohomology number H_{-3}. For the sake of completeness, let us recall some definitions made in [1]. The positive ghost cohomologies $H_n (n \geq 1)$ consist of physical operators existing at positive ghost pictures greater or equal to n, annihilated by Γ^{-1} at the minimal positive picture n. That is, the picture changing transformations (direct and inverse) allow to move the elements of H_n between pictures greater or equal to n, but not below n. Similarly, the negative ghost cohomologies, $H_{-n} (n \geq 3)$ consist of physical operators existing at ghost pictures $-n$ and below, while at their minimal negative picture $-n$ they are annihilated by the direct picture-changing Γ.

The zero cohomology H_0 by definition consists of picture-independent operators (i.e. standard perturbative vertex operators existing at all the ghost pictures). The cohomologies H_{-1} and H_{-2} are empty, since any operator at picture -1 or -2 is either BRST-exact or can be transformed to picture zero by Γ. The question of existence of nonzero ghost cohomologies (which by itself is quite non-trivial) has been discussed in details previously

[1]. The important property of the ghost cohomologies is the isomorphism between positive and negative H's:

$$H_n \sim H_{-n-2}$$

constructed in [1]. This isomorphism is *not* a picture changing equivalence, though it is somewhat reminiscent of it. Structurally, it maps cohomologies of positive and negative numbers, "bypassing" the pictures in the middle. Typically, commutators of the current algebra of operators from nonzero H's involve the currents from both negative and positive cohomologies, related by the isomorphism.

It is not difficult to show that, just like the operator (5) - the element of H_1 - generates the space-time symmetry transformations (3) of the RNS action (1), its picture -3 version (the element of H_{-3}) generates the symmetry transformations identical to (3), with e^{ϕ} replaced by $e^{-3\phi}$. For the critical ($d=10$) uncompactified RNS superstrings the set of the transformations (3) is the only additional nonlinear space-time symmetry. For non-critical strings ($d \neq 10$), however, there are $d+1$ additional α-symmetries, involving the Liouville sector. The appropriate transformations are given by

$$\begin{aligned} \delta X_m &= \epsilon_{m\alpha}\{\partial(e^{\phi}\lambda) + 2e^{\phi}\partial\lambda\} \\ \delta\lambda &= -\epsilon_{m\alpha}\{2\partial(e^{\phi}\partial X^m) + e^{\phi}\partial^2 X^m\} \\ \delta\gamma &= \epsilon_{m\alpha}e^{2\phi-\chi}\{\partial^2 X^m\lambda - 2\partial X^m\partial\lambda\} \\ \delta\beta &= \delta b = \delta c = \delta\varphi = \delta\psi^m = 0 \end{aligned} \tag{6}$$

and

$$\begin{aligned} \delta\varphi &= \epsilon_{-\alpha}\{\partial(e^{\phi}\lambda) + 2e^{\phi}\partial\lambda\} \\ \delta\lambda &= -\epsilon_{-\alpha}\{2\partial(e^{\phi}\partial\varphi) + e^{\phi}\partial^2\varphi\} \\ \delta\gamma &= \epsilon_{-\alpha}e^{2\phi-\chi}\{\lambda\partial^2\varphi - 2\partial\varphi\partial\lambda\} \\ \delta\beta &= \delta b = \delta c = \delta X^m = \delta\psi^m = 0 \end{aligned} \tag{7}$$

.

In the limit of zero cosmological constant, the dressed generators inducing the transformations (6) and (7) are given by [1]

$$\begin{aligned} L^{m\alpha} = l(d)\oint\frac{dz}{2i\pi}e^{\phi+Q\varphi}\{(\partial^2\varphi + Q(\partial\varphi)^2)\psi^m - 2\partial\varphi\partial\psi^m + \partial^2 X^m\lambda \\ -2\partial X^m\partial\lambda - 4Q\partial\varphi\lambda\partial X^m\} \end{aligned} \tag{8}$$

and

$$L^{-\alpha}=l(d)\oint\frac{dz}{2i\pi}e^{\phi+Q\varphi}\{(\partial^2\varphi+Q(\partial\varphi)^2)\lambda+\partial\varphi\partial\lambda\} \tag{9}$$

accordingly with the normalization constant $l(d)$ given by [2],[1]

$$l(d)=\sqrt{\frac{k(d)+1}{k(d)+2}} \tag{10}$$

with

$$k(d)=\frac{1}{8}(d-1\pm\sqrt{(d-1)(d-9)}) \tag{11}$$

The generators (8) and (9) are the Virasoro primaries, annihilated by the inverse picture changing. They are BRST non-trivial and invariant (upon adding the $b-c$ ghost correction terms which we have skipped) and therefore are the elements of $H_1 \sim H_{-3}$. As before, the H_{-3} version of the generators (8), (9) with the manifest BRST-invariance (with no $b-c$ correction terms) and the set of the related space-time transformations can be obtained simply by replacing $\phi\rightarrow -3\phi$ in (6) - (9). Combined with $\frac{(d+1)(d+2)}{2}$ dimension 1 Virasoro primaries:

$$\begin{aligned}L^{mn}&=\oint\frac{dz}{2i\pi}\psi^m\psi^n\\ L^{+m}&=\oint\frac{dz}{2i\pi}e^{-\phi}\psi^m\\ L^{-m}&=l(d)\oint\frac{dz}{2i\pi}e^{Q\varphi}\psi^m\lambda\\ L^{++}&=l(d)\oint\frac{dz}{2i\pi}e^{-\phi+Q\varphi}\lambda,\end{aligned} \tag{12}$$

that induce $d+1$ translations and $\frac{d(d+1)}{2}$ rotations in space-time (including the Liouville direction), the $d+2$ currents (5), (8), (9) of $H_1 \sim H_{-3}$ enlarge the current algebra of the space-time symmetry generators from $SO(d,2)$ to $SO(d+1,2)$, effectively bringing in extra dimension to the theory. Indeed, introducing the $d+2$-dimensional index $M=(m,+,-,\alpha); m=0,...d-2;\alpha=1$ with the $(d,2)$ metric η^{MN} consisting of $\eta^{mn},\eta^{+-}=-1,\eta^{--}=\eta{++}=0,\eta^{\alpha\alpha}=1$ and evaluating the commutators of the operators (5), (8), (9), (12) it isn't difficult to check that [1]

$$[L^{M_1N_1},L^{M_2N_2}]=\eta^{M_1M_2}L^{N_1N_2}+\eta^{N_1N_2}L^{M_1M_2}-\eta^{M_1N_2}L^{M_2N_1}-\eta^{M_2N_1}L^{M_1N_2} \tag{13}$$

Note that, as the $SO(d,2)$ space-time symmetry group of translations and rotations for non-critical RNS strings is identical to the isometry group of the AdS_d space, the

$H_1 \sim H_{-3}$ generators (5), (8), (9) of the lowest nonzero ghost cohomology are simply the stringy analogues of the off-shell symmetry generators from hidden space-time dimension, observed in the 2T physics approach [3], [4], [5] for a particle in the AdS_d space [3]. The precise correspondence between the space-time symmetry generators for a AdS_d particle in the 2T physics formalism (including the extra generators from hidden dimension) and the space-time symmetry generators for non-critical strings(including $H_1 \sim H_{-3}$ L-generators of α-symmetries) is given by

$$\begin{aligned} L^{mn} &\leftrightarrow T^{mn} \\ L^{+m} \leftrightarrow T^{+m}, L^{-m} &\leftrightarrow T^{-m} \\ L^{++} &\leftrightarrow T^{+-} \end{aligned} \tag{14}$$

for the "standard" generators of the $SO(d,2)$ subgroup of $SO(d,2)$ and

$$\begin{aligned} L^{+\alpha} \leftrightarrow T^{+}, L^{-\alpha} &\leftrightarrow T^{-} \\ L^{m\alpha} &\leftrightarrow T^{m} \end{aligned} \tag{15}$$

for the extra generators (associated with the higher space-time dimension). Here the symmetry generators for the AdS_d particle are given by (using the notations of [3]):

$$\begin{aligned} T^{mn} = x^m p^n - x^n p^m, T^{+m} = p^m, T^{+-} &= -uk + (\vec{x}\vec{p}) \\ T^{-m} &= \frac{p^m}{2u^2} + ukx^m + \frac{1}{2}x^2 p^m - (\vec{x}\vec{p})x^m \\ T^{-} &= \frac{1}{2}k + \frac{1}{2}x^2(u^2 k - 2ku^2) - \frac{(\vec{x}\vec{p})}{u} \\ T^{+} = -u^2 k - 2ku^2, T^{m} &= x^m(u^2 k - 2ku^2) - \frac{p^m}{u} \end{aligned} \tag{16}$$

where the AdS_d metric is given by

$$\begin{aligned} ds^2 &= u^2 (dx^m)^2 + \frac{du}{u^2} \\ m &= 0, 1, ..., d-2 \end{aligned} \tag{17}$$

and p^m and k are the canonical conjugates of x^m and u respectively. Here u is the radial AdS_d coordinate, corresponding to the Liouville direction on the string theory side. Thus the picture-dependent generators of $H_1 \sim H_{-3}$ of the lowest nonzero ghost cohomology are in one to one correspondence with the symmetry generators from hidden space-time dimension in the $2T$ approach. In the following section we shall discuss the generalization of this result to ghost cohomologies of higher orders.

2. α-Symmetries at the level $H_2 \sim H_{-4}$

The natural question is how to extend the results, described in the previous section, to higher order cohomologies, i.e. the BRST-invariant primaries from $H_2 \sim H_{-4}, H_3 \sim H_{-5}$, etc. Do these currents generate more space-time symmetries? If yes, are these new symmetries related to yet unknown higher space-time dimensions, not detected in the $2T$ formalism? Can these extra dimensions be understood in terms of the class of holography relations, so that each extra dimension is effectively generated by operators from the associate ghost cohomology?

In the rest of the paper, we will try to address these questions. It is instructive to start from the simplest example of non-critical RNS strings - the supersymmetric $c = 1$ model where the straightforward construction of the elements of $H_n \sim H_{-n-2}$ is relatively simple. It has been shown [6] that the physical operators from the higher ghost cohomologies enlarge the current algebra (which becomes the target space symmetry of the theory when the space dimension is compactified at the self-dual radius).That is, while the original current algebra of the theory is given by SU(2) [7], [8], [9], so the standard picture-independent discrete states are the SU(2) multiplets, introducing the target space symmetry generators of higher cohomologies of ghost numbers up to N (i.e. extending the current algebra with the elements of $H_n \sim H_{-n-2}$ with $n = 1, 2, ...N$) enhances the symmetry algebra of the $c = 1$ theory (compactified at self-dual radius) from $SU(2)$ to $SU(N+2)$. Let us review first the simplest case of $N = 1$. One starts with the $H_1 \sim H_{-3}$ generator

$$T_{-3,2} = \oint \frac{dz}{2i\pi} e^{-3\phi+2iX}\psi \tag{18}$$

with momentum $+2$ in the X-direction (the left index refers to the ghost cohomology number and the right one to the momentum) and acts on it repeatedly with the lowering operator $T_{0,-1} = \oint \frac{dz}{2i\pi} e^{-iX}\psi$ of $SU(2)$, obtaining altogether five $H_{-3} \sim H_1$ generators with the discrete momenta$-2 \leq p \leq 2$ (note that the momentum -2 generator is annihilated by $T_{0,-1}$). Unified with 3 currents of $SU(2)$, 5 currents of $H_1 \sim H_{-3}$ combine into 8 operators generating $SU(3)$ [6]. The lowering subalgebra of $SU(3)$ consists of 3 operators with negative discrete momenta (one from H_0 and two from $H_1 \sim H_{-3}$), the upper subalgebra includes those with the positive momenta while two zero momentum generators $T_{00} \sim \oint \frac{dz}{2i\pi}\partial X$ of H_0 and

$$T_{-3,0} = \oint \frac{dz}{2i\pi} e^{-3\phi}(\partial^2 X\psi - 2\partial X\partial\psi) \tag{19}$$

generate the Cartan subalgebra. This construction also can be generalized for the case of arbitrary $H_N \sim H_{-N-2}$ [6]. To construct the generators of $H_N \sim H_{-N-2}$ one starts with $T_{-N-2,N+1} = \oint \frac{dz}{2i\pi} e^{-(N+2)\phi + i(N+1)X} \psi$ and acts repeatedly with $T_{0,-1}$, obtaining $2N+3$ generators with the discrete momenta $-N-1 \leq p \leq N+1$. Unifying all the operators of $H_n \sim H_{-n-2}, 0 \leq n \leq N$, obtained by this procedure, one combines them into $(N+2)^2 - 1$ generators of $SU(N+2)$. Among these generators there are $N+1$ currents with the zero momentum that form the Cartan subalgebra of $SU(N+2)$ (all the Cartan generators appear from different ghost cohomologies). Acting with the lowering generators of $SU(N+2)$ on the highest weight vectors (which typically are the tachyonic primaries at integer momenta) one generates the extended physical spectrum of picture-dependent physical states - the SU(N+2) multiplets. This has been shown explicitly for $N=1$ and $N=2$ and conjectured for higher values of N. Accordingly, the structure constants of the vertex operators are given by the $SU(N+2)$ Clebsch-Gordan coefficients and thus the operator algebra is isomorphic to the volume-preserving diffeomorphisms in $N+2$ dimensions [6], [8]. The natural interpretation of this result is that each separate nonzero ghost cohomology brings in an effective extra dimension and this implies the holographic correspondence between $c=1$ supersymmetric model, including the states from nonzero H_n, to field theories in higher dimensions. As we have seen before, higher dimensional RNS models follow the same pattern for $N=1$, as the currents (5), (8), (9) of $H_1 \sim H_{-3}$ increase the dimensionality of the space-time symmetry group by 1 unit. There is, however, an important difference. The case of $c=1$ compactified at the self-dual radius, is special as the symmetry generators include operators at nonzero momenta. The uncompactified non-critical strings in $d>1$, however, cannot contain generators at non-zero momenta since in operators of the type $\sim e^{ikX}$ the left and right modes aren't separated and such generators generally cannot be integrated over a worldsheet contour. For this reason, the only $H_n \sim H_{-n-2}$ generators that can be exported from the $c=1$ case to higher dimensions are those with zero momentum, i.e. the Cartan generators of $SU(n+2)$. For example, the Liouville-independent part of the $H_1 \sim H_{-3}$ generator of α-symmetry (5) has the structure identical to the Cartan of $SU(3)$ (that can be interpreted as a "hypercharge operator" [6]) For large values of n the expressions for Cartan generators of $SU(n+2)$ are more complicated [6], however, the relation to Cartan generators is still useful in order to to conjecture some properties of $H_n \sim H_{-n-2}$ generators to simplify our searches. In particular, it is quite clear that

1)expressions for the $H_n \sim H_{-n-2}$ generators do not contain any derivatives of the superconformal ghost field ϕ, so their ghost dependence is determined entirely by $e^{n\phi}$ or $e^{-(n+2)\phi}$

2) the matter parts of $SU(n)$ Cartan generators of $c = 1$ can be used as building blocks to construct the higher cohomolody elements for non-critical strings in higher space-time dimensions.

Given these indications, below we shall look for the elements of $H_2 \sim H_{-4}$ generating the higher order α-transformations. To avoid technical complications with inverse picture-changing due to the $b - c$ ghost terms in the H_2 representation, we shall concentrate on the H_{-4}-version of the generators. We start from the Liouville-independent part of the α-transformations (i.e. those that mixes matter with ghosts but doesn't touch the Liouville sector). In general, these cohomology elements - the candidates for the α-symmetry generators - could be in various tensor representations of the Lorenz group, so we start with the simplest case of the scalar generator. The most general expression for a scalar dimension 1 operator of the ghost number -4, in view of the above conditions, is given by

$$
\begin{aligned}
V(z) = e^{-4\phi}\{&\alpha_1(\partial X_m \partial X^m)(\partial X^n \partial^2 X^n) + \alpha_2(\partial X_m \partial X^m)(\psi_n \partial^2 \psi^n) \\
&+\alpha_3(\partial X_m \partial^2 X^m)(\psi_n \partial \psi^n) + \alpha_4(\psi_m \partial \psi^m)(\psi_n \partial^2 \psi^n) + \beta_1(\psi_m \partial X^m)(\partial X_n \partial^2 \psi^n) \\
&+\beta_2(\psi_m \partial X^m)(\partial^2 X_n \partial \psi^n) + \beta_3(\psi_m \partial X^m)(\psi_n \partial^3 X^n) + \beta_4(\psi_m \partial^2 X^m)(\partial \psi_n \partial X^n) \\
&+\gamma_1(\partial X_m \partial^4 X^m) + \gamma_2(\partial^2 X_m \partial^3 X^m) + \lambda_1(\psi_m \partial^4 \psi^m) + \lambda_2(\partial \psi_m \partial^3 \psi^m)\}(z)
\end{aligned}
\tag{20}
$$

where $\alpha_i, \beta_i, \gamma_i$ and λ_i are some numbers. In order to be the element of H_{-4}, $V(z)$ of (20) must satisfy two necessary conditions:

1) it must be a primary field, i.e. its OPE with the full matter+ghost stress-energy tensor must not contain singularities of the order higher than $(z-w)^{-2}$, i.e. all the OPE coefficients in front of the terms of the order of $(z-w)^{-n}, n \geq 3$ have to vanish separately.

2) it must be annihilated by the picture-changing operator, i.e. its OPE with Γ can only contain terms of the order of $(z-w)^n, n \geq 1$, that is, the coefficients in front of the lower order terms must all vanish separately.

These two conditions altogether generate a number of linear constraints on the $\alpha_i, \beta_i, \gamma_i$ andλ_i coefficients. Any combination of the coefficients, satisfying these constraints (in addition to the BRST triviality condition), is a candidate for the element of H_{-4} and for the higher order α-symmetry generator. Note that the primary field + annihilation constraints are necessary, but generally not sufficient conditions to define the α-generator

since the latter also has to be BRST-nontrivial, as BRST-exact currents obviously can't generate any new global space-time symmetries. The non-triviality needs be checked separately or alternatively, one has to verify straightforwardly that the transformations induced by the candidate operator, are indeed the symmetries of the RNS action.

We start with the primary field constraints on V. Using the matter+ghost stress-energy tensor:

$$T(z) = -\frac{1}{2}\partial X_m \partial X^m - \frac{1}{2}\partial\psi_m\psi^m - \frac{1}{2}(\partial\varphi)^2 + \frac{Q}{2}\partial^2\varphi + T_{b-c} + T_{\beta\gamma} \tag{21}$$

the straightforward calculation gives

$$\begin{aligned} T(z)V(w) = (z-w)^{-7}(48\lambda_1 + 6\lambda_2 - 24\gamma_1 - 12\gamma_2) \\ +(z-w)^{-5}\{(2\beta_2 + 2d\alpha_3 - 12\lambda_1 + 12\lambda_2 + 2(1-d)\alpha_4)\partial\psi_m\psi^m \\ +(2d\alpha_2 - 2d\alpha_1 + 2\beta_1 + 24\gamma_1)\partial X_m\partial X^m\} \\ +(z-w)^{-4}\{(\frac{1}{2}(1-d)\alpha_4 + \beta_1 + d\alpha_2)\partial^2\psi_m\psi^m \\ +(\frac{1}{2}(d\alpha_3 + \alpha_4 + \beta_2 + \beta_4) - (d+2)\alpha_1 + 6\gamma_2)\partial X_m\partial^2 X^m\} \\ +(z-w)^{-3}\{\lambda_2\psi_m\partial^3\psi^m + 2\gamma_2(\partial X_m\partial^3 X^m) + 2\alpha_1(\partial X_m\partial X^m)^2 \\ +\alpha_4(\partial\psi_m\psi^m)^2 + (2\alpha_3 + \alpha_2)(\partial X_m\partial X^m)(\psi_m\partial\psi^m) \\ +(\beta_1 + 2\beta_2 + 2\beta_4)(\psi_m\partial X^m)(\partial\psi_m\partial X^m) \\ +(\beta_2 - \beta_4)(\psi_m\partial X^m)(\psi_m\partial^2 X^m)\} + O((z-w)^{-2}) \end{aligned} \tag{22}$$

,

so the primary field constraints are

$$
\begin{aligned}
48\lambda_1 + 6\lambda_2 - 24\gamma_1 - 12\gamma_2 &= 0 \\
2\beta_2 + 2d\alpha_3 - 12\lambda_1 + 12\lambda_2 + 2(1-d)\alpha_4 &= 0 \\
2d\alpha_2 - 2d\alpha_1 + 2\beta_1 + 24\gamma_1 &= 0 \\
\frac{1}{2}(1-d)\alpha_4 + \beta_1 + d\alpha_2 &= 0 \\
\frac{1}{2}(d\alpha_3 + \alpha_4 + \beta_2 + \beta_4) - (d+2)\alpha_1 + 6\gamma_2 &= 0 \\
\lambda_2 = \gamma_2 = \alpha_1 = \alpha_4 &= 0 \\
2\alpha_3 + \alpha_2 &= 0 \\
\beta_1 + 2\beta_2 + 2\beta_4 &= 0 \\
\beta_2 - \beta_4 &= 0
\end{aligned}
\tag{23}
$$

Next, consider the annihilation condition $\Gamma V \sim 0$. Since V is the dimension 1 integrand of vertex operator in the integrated form, the $c\partial\xi$ term of Γ of fermionic ghost number 1 doesn't act on V while the $b-c$ ghost number -1 term $\sim be^{2\phi-\chi}(\partial\chi + \partial\sigma)$ annihilates V for any choice of the coefficients α, β, γ and λ. Therefore it is sufficient to consider how V is acted on by the $b-c$ ghost number zero term $\sim e^{\phi}G_{matter} \sim e^{\phi}\psi_m\partial X^m$. Note that, as the expression for V is Liouville-independent, the OPE of V with $e^{\phi}G_{Liouville}$ vanishes as well. The OPE evaluation of Γ and V gives

$$
\begin{aligned}
e^{\phi}\psi_m\partial X^m(z)V(w) &= ((z-w)^{-1} + \partial\phi(z-w)^0)e^{-3\phi}\psi_m\partial X^m(w) \\
\times\{ -4\alpha_2 - 2\alpha_3 - 2(d+1)\beta_1 &- 2d\beta_2 + 6(1-d)\beta_3 - 2\beta_4 - 24\gamma_1 + 24\lambda_1\} \\
+(z-w)^0\{e^{-4\phi}\partial X_m\partial\psi^m(w)&(2\alpha_3 + 2\beta_2 + 2d\beta_4 + 6\lambda_2 - 24\gamma_1) \\
+e^{-4\phi}\partial^2 X_m\psi^m(w)\{ -2\alpha_3 &- \beta_2 - d\beta_4 - 6\gamma_2 + 24\gamma_1\} + O(z-w)
\end{aligned}
\tag{24}
$$

accordingly the annihilation constraints are

$$
\begin{aligned}
-4\alpha_2 - 2\alpha_3 - 2(d+1)\beta_1 - 2d\beta_2 + 6(1-d)\beta_3 - 2\beta_4 - 24\gamma_1 + 24\lambda_1 &= 0 \\
2\alpha_3 + 2\beta_2 + 2d\beta_4 + 6\lambda_2 - 24\gamma_1 &= 0 \\
-2\alpha_3 - \beta_2 - d\beta_4 - 6\gamma_2 + 24\gamma_1 &= 0
\end{aligned}
\tag{25}
$$

Primary field constraints (22) along with the annihilation constraints (23) define the elements of the cohomology (provided they are BRST non-trivial). Simple check shows that the system of linear equations (22), (23) has no nonzero solutions, therefore $H_{-4} \sim H_2$ does not contain any Liouville-independent scalars.

Now consider the Liouville-independent vector candidates for H_{-4}. The analysis, presented in the Appendix of this paper, is analogous to the scalar case and, as before, we find no Liouville-independent BRST non-trivial vector operators of $H_{-4} \sim H_2$ generating any new symmetries of the RNS action. Similarly, one can show that $H_{-4} \sim H_2$ contains no higher rank tensors generating symmetries in the space-time. Therefore, unlike the $H_{-3} \sim H_1$ case (with one Liouville-independent generator), all the α-symmetry generators of $H_2 \sim H_{-4}$ depend on the Liouville mode. Our aim is now to find the Liouville-dependent currents of $H_2 \sim H_{-4}$. Instead of the straightforward procedure described above (i.e. solving the annihilatilation + primary field constraints for a general picture -4 operator), involving lengthy calculations, it is easier to guess the structure of the cohomology elements and then to verify directly that they satisfy the operator algebra generating a space-time symmetry with an extra dimension and induce the space-time symmetry transformation. For simplicity, in the rest of the paper we will restrict ourselves to the case of the zero dilaton field so we can neglect the background charge and the related effects of the second derivative term in the Liouville stress tensor. In principle all our calculations can be straightforwardly generalized to the case of non-zero dilaton as well to account for the effect of the dressing (as it has been done in (8), (9), (12) for the case of $H_1 \sim H_{-3}$), however the relevant expressions for the generators become quite cumbersome and such complications aren't necessary for our purposes.

With the dimension $\frac{5}{2}$ scalar primary fields

$$\begin{aligned} F_1(X,\psi) &= \partial^2 X_m \psi^m - 2\partial X_m \partial\psi^m \\ F_1(\varphi,\lambda) &= \partial^2\varphi\lambda - 2\partial\varphi\partial\lambda \end{aligned} \tag{26}$$

it is not difficult to check that the generator

$$T = \oint \frac{dz}{2i\pi} e^{-4\phi}(\partial^2 X_m\psi^m - 2\partial X_m\partial\psi^m)(\partial^2\varphi\lambda - 2\partial\varphi\partial\lambda) \tag{27}$$

is in the cohomology, i.e. is the scalar element of H_{-4}. Note that $F_1(X,\psi)$ has the structure of the matter part of the Cartan generator of $SU(3)$ (27) while $F_1(\varphi,\lambda)$ is its Liouville copy, with the $c=1$ matter fields X and ψ replaced with φ and λ. Similarly to (13), it is now appropriate to introduce the $d+4$-dimensional space-time index $M = (m,+,-.\alpha,\beta)$ with $m = 0,...d-1; \alpha,\beta = 1$, and the $(m,+,-)$ indices correspond to $SO(d,2)$ isometry group of AdS_{d+1} generated by the matter+Liouville fields. As before, the index $\alpha = 1$ labels the extra space-time dimension generated by the currents of $H_{-3} \sim H_1$ (that enlarge

the AdS_{d+1} isometry with the α-symmetry; as the new index $\beta = 1$, it will correspond to the new extra dimension induced by $H_{-4} \sim H_2$ currents of the higher order α-symmetry. As we expect the complete space-time symmetry group to be extended from $SO(d+1,2)$ to $SO(d+2,2)$ by the operators of $H_2 \sim H_{-4}$, we look for $d+3$ generators $L^{m\beta}$, $L^{\beta+}$, $L^{\beta-}$ and $L^{\alpha\beta}$ of $H_{-4} \sim H_2$, i.e. for 1 space-time d-vector and 3 scalars. The first step is to correctly identify the operator (27) with one of 3 $H_{-4} \sim H_2$ scalar generators of $SO(2, d+2)$. In particular, these generators have to satisfy

$$\begin{aligned}
[L^{\beta\alpha}, L^{+m}] = [L^{\beta+}, L^{-m}] = 0; \\
[L^{\beta-}, L^{+m}] = L^{\beta+}; [L^{\beta+}, L^{-m}] = L^{\beta m}; \\
[L^{\beta\alpha}, L^{++}] = [L^{\beta+}, L^{++}] = 0; \\
[L^{\beta-}, L^{++}] = L^{\beta+}.
\end{aligned} \tag{28}$$

On the other hand, straightforward evaluation of the commutators of (28) with the generators of the SO(2,d) subalgebra gives

$$\begin{aligned}
[T, L^{-m}] = \oint \frac{dz}{2i\pi} e^{-4\phi} \{ & (\partial^2 X^m \lambda - 2\partial X^m \partial\lambda)(\partial^2\varphi\lambda - 2\partial\varphi\partial\lambda) \\
& + (\partial^2 X_n \psi^n - 2\partial X_n \partial\psi^n)(-\partial^2\varphi\psi^m + 2\partial\varphi\partial\psi^m)\}; \\
& [T, L^{++}] = [T, L^{+m}] = 0.
\end{aligned} \tag{29}$$

Comparing (28) and (29) it is clear that the T-operator (27) must be identified with $L^{\beta+}$. In addition, the comparison of (28) and (29) also allows to deduce expressions for the d-vector generator of $H_2 \sim H_{-4}$, namely,

$$\begin{aligned}
L^{\beta m} = \oint \frac{dz}{2i\pi} e^{-4\phi} \{ & (\partial^2 X^m \lambda - 2\partial X^m \partial\lambda)(\partial^2\varphi\lambda - 2\partial\varphi\partial\lambda) \\
& + (\partial^2 X_n \psi^n - 2\partial X_n \partial\psi^n)(-\partial^2\varphi\psi^m + 2\partial\varphi\partial\psi^m)\}
\end{aligned} \tag{30}$$

Next, evaluating the commutator of $L^{\beta m}$ with the genarator L^{-n} of the rotations in the matter-Liouville planes, we deduce the second scalar generator $L^{\beta-}$:

$$\begin{aligned}
[L^{\beta m}, L^{-n}] = \eta^{mn} L^{\beta-} \\
L^{\beta-} = \oint \frac{dz}{2i\pi} e^{-4\phi} (\partial^2 X_l \lambda - 2\partial X_l \partial\lambda)(2\partial\varphi\partial\psi^l - \partial^2\varphi\psi^l)
\end{aligned} \tag{31}$$

It is now quite straightforward to obtain the final, third scalar generator of $L^{\alpha\beta}$ of $H_2 \sim H_{-4}$. In particular it can be deduced from the commutator $[L^{\beta-}, L^{\alpha+}] = -\eta^{+-} L^{\alpha\beta}$. The

computation is somewhat lengthy as $L^{\beta-}$ is taken at picture -4 and $L^{\alpha+}$ at picture -3, so the obtained picture -7 generator must be transformed back to picture -4 by using the direct picture-changing operator : $\Gamma := \frac{i}{\sqrt{2}}(\psi_m \partial X^m + \lambda\partial\varphi) + ...$ (with insignificant $b-c$ ghost terms ignored). The result is given by

$$
\begin{aligned}
L^{\alpha\beta} = \frac{i}{\sqrt{2}}\oint \frac{dz}{2i\pi} e^{-4\phi}\{\frac{1}{4}(\partial\varphi)^5 - \frac{3}{4}\partial\varphi(\partial^2\varphi)^2 + \frac{1}{4}(\partial\varphi)^2\partial^3\varphi + \lambda\partial\lambda(\partial^3\varphi - (\partial\varphi)^3) \\
-\frac{3}{2}\lambda\partial^2\lambda\partial^2\varphi + 3\partial\lambda\partial^2\lambda\partial\varphi\} \equiv \oint \frac{dz}{2i\pi} e^{-4\phi} F_2(\varphi,\lambda) \equiv: (\oint e^{-i\varphi}\lambda)^3 \oint e^{-4\phi+3i\varphi}\lambda :
\end{aligned}
\quad (32)
$$

where, for convenience, we have denoted the matter part of $L^{\alpha\beta}$ by F_2 (c.f. F_1, the matter part of the $H_1 \sim H_{-3}$ Cartan generator (19) of $SU(3)$).

Not surprisingly, this generator is just the $H_{-4} \sim H_2$ Cartan generator of $SU(4)$ of the supersymmetric $c = 1$ model [6] with the matter fields X and ψ of the $c = 1$ theory replaced with φ and λ (recall that we neglect the effect of the dilaton field and the related background charge). As has been explained in [6] this generator can be obtained by taking the H_{-4} generator $\oint \frac{dz}{2i\pi} e^{-4\phi+3iX}\psi$ carrying the discrete $+3$ momentum and applying to it the SU(2) lowering generator $\sim \oint \frac{dz}{2i\pi} e^{-iX}\psi$ three times. This constitutes the complete set of the $H_{-4} \sim H_2$ generators. It is now easy to compute the rest of the commutators involving the $d+3$ generators of $H_{-4} \sim H_2$: $(L^{\beta m}, L^{\beta+}, L^{\beta-} L^{\beta\alpha})$ of (27)-(32) and to show that, combined with $d+2$ generators $(L^{\alpha m}, L^{\alpha+}, L^{\alpha-})$ of $H_{-3} \sim H_1$ of (5),(8),(9) and $\frac{(d+1)(d+2)}{2}$ isometry generators (12) (disregarding the Liouville dressing) of AdS_{d+1}, constitute $\frac{(d+3)(d+4)}{2}$ generators of $SO(2, d+2)$ satisfying the algebra (13), but with the capital indices M, N of (13) being now $d+4$-dimensional: $M = (m, +, -, \alpha, \beta)$. Therefore, just like in the case of $H_1 \sim H_{-3}$, the generators of $H_2 \sim H_{-4}$ descend from the hidden extra dimension associeted with this cohomology. The total number of extra dimensions (apart from the Liouville direction) now equals 2, in accordance with the concept relating each ghost cohomology (particularly, $H_1 \sim H_{-3}$ and $H_2 \sim H_{-4}$) to an associate space-time dimension. The remaining step is to show that the $H_2 \sim H_{-4}$ generators $L^{\beta m}, L^{\beta+}, L^{\beta-}$ and $L^{\beta\alpha}$ of $SO(2, d+2)$ are indeed the space-time symmetry generators, i.e. they induce $d+3$ higher order α symmetries originating from the new space-time dimension associated with the higher order ghost cohomology $H_2 \sim H_{-4}$. Using the $H_2 \sim H_{-4}$ generators (27)-(32), it is not difficult to derive the space-time transformations induced by (27) and (29) - (32). We start from the first set of the $H_2 \sim H_{-4}$ related α-transformations induced by $L^{\beta+} = e^{2\phi} F_1(X, \psi) F_1(\varphi, \lambda)$.

Just like in the $H_1 \sim H_{-3}$ case, we choose to work with the positive picture $+2$ representation of the L^β-operators (the negative -4 picture represenation results in the equivalent set of transformations; when applied to the RNS Lagrangian, the difference results only in the total derivative terms). Simple calculation shows that these transformations are given by

$$
\begin{aligned}
\delta X^n &= \epsilon^{\beta+}\{2e^{2\phi}\partial\psi^n(\partial^2\varphi\lambda - 2\partial\varphi\partial\lambda) + \partial(e^{2\phi}\psi^n(\partial^2\varphi\lambda - 2\partial\varphi\partial\lambda))\} \\
\delta\psi^n &= \epsilon^{\beta+}\{-e^{2\phi}\partial^2 X^n(\partial^2\varphi\lambda - 2\partial\varphi\partial\lambda) - \partial(e^{2\phi}\partial X^n(\partial^2\varphi\lambda - 2\partial\varphi\partial\lambda))\} \\
\delta\varphi &= \epsilon^{\beta+}\{e^{2\phi}(\partial^2 X_m\psi^m - 2\partial X_m\partial\psi^m)\partial\lambda + \partial(e^{2\phi}\lambda(\partial^2 X_m\psi^m - 2\partial X_m\partial\psi^m))\} \\
\delta\lambda &= \epsilon^{\beta+}\{-e^{2\phi}(\partial^2 X_m\psi^m - 2\partial X_m\partial\psi^m)\partial^2\varphi - 2\partial(e^{2\phi}(\partial^2 X_m\psi^m - 2\partial X_m\partial\psi^m)\partial\varphi)\} \\
\delta\gamma &= \epsilon^{\beta+}e^{3\phi-\chi}\{2\partial\phi F_1(X,\psi)F_1(\varphi,\lambda) + \partial(F_1(X,\psi)F_1(\varphi,\lambda))\} \\
\delta\beta &= \delta b = \delta c = 0
\end{aligned}
\tag{33}
$$

where $\epsilon^{\beta+}$ is transformation parameter. For technical reasons, it is also convenient to write explicitly the transformations for γ in terms of the bosonized fields ϕ and χ:

$$
\begin{aligned}
\delta(\partial\phi) &= 2\epsilon^{\beta+}e^{2\phi}F_1(X,\psi)F_1(\varphi,\lambda) \\
\delta\chi &= 0
\end{aligned}
\tag{34}
$$

It is now easy to check that the transformations (33),(34) leave the RNS action invariant, i.e. generate the global space-time symmetry. Applying the transformations (33),(34) to the action (1) and using some simple integration by parts it is straightforward to show that

$$
\begin{aligned}
\delta S_{ghost} &= \frac{\epsilon^{\beta+}}{\pi}\int d^2z(\bar\partial e^{2\phi})F_1(X,\psi)F_1(\lambda,\varphi) \\
\delta S_{matter} \equiv \delta(S_{X,\psi} + S_{Liouville}) &= -\frac{\epsilon^{\beta+}}{\pi}\int d^2z(\bar\partial e^{2\phi})F_1(X,\psi)F_1(\varphi,\lambda)
\end{aligned}
\tag{35}
$$

so the α-variation of the matter part is precisely cancelled by that of the ghost part, just like in the $H_1 \sim H_{-3}$ case. The transformations (33),(34) thus constitute the first set of the higher order $H_2 \sim H_{-4}$ α-symmetries. The second set, induced by $L^{\beta-}$, is given by

$$\begin{aligned}
\delta X^n &= \epsilon^{\beta-}\{2e^{2\phi}\partial\lambda(2\partial\varphi\partial\psi^n - \partial^2\varphi\psi^n) + \partial(e^{2\phi}\lambda(2\partial\varphi\partial\psi^n - \partial^2\varphi\psi^n))\} \\
\delta\psi^n &= \epsilon^{\beta-}\{e^{2\phi}(\partial^2X^n\lambda - 2\partial X^n\partial\lambda)\partial^2\varphi + 2\partial(e^{2\phi}(\partial^2X^n\lambda - 2\partial X^n\partial\lambda)\partial\varphi)\} \\
\delta\varphi &= -2\epsilon^{\beta-}\}e^{2\phi}(\partial^2X_n\lambda - 2\partial X_n\partial\lambda)\partial\psi^n - \partial(e^{2\phi}(\partial^2X_n\lambda - 2\partial X_n\partial\lambda)\psi^n)\} \\
\delta\lambda &= \epsilon^{\beta-}\{-e^{2\phi}\partial^2X_n(2\partial\varphi\partial\psi^n - \partial^2\varphi\psi^n) - 2\partial(e^{2\phi}\partial X_n(2\partial\varphi\partial\psi^n - \partial^2\varphi\psi^n))\} \\
\delta(\partial\phi) &= 2\epsilon^{\beta-}e^{2\phi}(\partial^2X_n\lambda - 2\partial X_n\partial\lambda)(2\partial\varphi\partial\psi^n - \partial^2\varphi\psi^n) \\
\delta\chi &= \delta b = \delta c = 0
\end{aligned} \tag{36}$$

The third set of d space-time α-transformations, induced by the d-vector generator $L^{\beta m}$ of $H_2 \sim H_{-4}$, is given by

$$\begin{aligned}
\delta X^n &= \epsilon^{\beta n}\{2e^{2\phi}\partial\lambda(\partial^2\varphi\lambda - 2\partial\varphi\partial\lambda) + \partial(e^{2\phi}\lambda(\partial^2\varphi\lambda - 2\partial\varphi\partial\lambda))\} \\
&+\epsilon^{\beta m}\{2e^{2\phi}\partial\psi^n(-\partial^2\varphi\psi_m + 2\partial\varphi\partial\psi_m) + \partial(e^{2\phi}\psi^n(-\partial^2\varphi\psi_m + 2\partial\varphi\partial\psi_m))\} \\
\delta\psi^n &= \epsilon^{\beta n}\{e^{2\phi}\partial^2\varphi(\partial^2X_m\psi^m - 2\partial X_m\partial\psi^m) + 2\partial(e^{2\phi}\partial\varphi(\partial^2X_m\psi^m - 2\partial X_m\partial\psi^m))\} \\
&+\epsilon^{\beta m}\{e^{2\phi}\partial^2X^n(-\partial^2\varphi\psi_m + 2\partial\varphi\partial\psi_m) + 2\partial(e^{2\phi}\partial X^n(-\partial^2\varphi\psi_m + 2\partial\varphi\partial\psi_m))\} \\
\delta\varphi &= \epsilon^{\beta n}\{-e^{2\phi}\partial\lambda(\partial^2X_n\lambda - 2\partial X_n\partial\lambda) - \partial(e^{2\phi}\lambda(\partial^2X_n\lambda - 2\partial X_n\partial\lambda)) \\
&+2e^{2\phi}\partial\psi_n(\partial^2X_m\psi^m - 2\partial X_m\partial\psi^m) + \partial(e^{2\phi}\psi_n(\partial^2X_m\psi^m - 2\partial X_m\partial\psi^m))\} \\
\delta\lambda &= \epsilon^{\beta n}\{-e^{2\phi}\partial^2\varphi(\partial^2X_n\lambda - 2\partial X_n\partial\lambda) - 2\partial(e^{2\phi}\partial\varphi(\partial^2X_n\lambda - 2\partial X_n\partial\lambda)) \\
&+e^{2\phi}\partial^2X_n(\partial^2\varphi\lambda - 2\partial\varphi\partial\lambda) + 2\partial(e^{2\phi}\partial X_n(\partial^2\varphi\lambda - 2\partial\varphi\partial\lambda))\} \\
\delta(\partial\phi) &= 2\epsilon^{\beta n}\{(\partial^2X_n\lambda - 2\partial X_n\partial\lambda)(\partial^2\varphi\lambda - 2\partial\varphi\partial\lambda) \\
&+(\partial^2X_m\psi^m - 2\partial X_m\partial\psi^m)(-\partial^2\varphi\psi_n + 2\partial\varphi\partial\psi_n)\} \\
\delta\chi &= \delta b = \delta c = 0
\end{aligned} \tag{37}$$

Finally, the transformations induced by $L^{\beta\alpha}$ are given by

$$\delta\varphi = \epsilon^{\beta\alpha}\{-\frac{5}{4}e^{2\phi}(\partial\varphi)^4 + \frac{3}{4}e^{2\phi}(\partial^2\varphi)^2 - \frac{3}{2}\partial(e^{2\phi}\partial\varphi\partial^2\varphi) - \frac{3}{4}e^{2\phi}(\partial\varphi)^2\partial^3\varphi - \frac{1}{4}\partial^2(e^{2\phi}(\partial\varphi)^2)$$
$$+\partial^2(e^{2\phi}(\lambda\partial\lambda)) - 3e^{2\phi}\lambda\partial\lambda(\partial\varphi)^2 - \frac{3}{2}\partial(e^{2\phi}\lambda\partial^2\lambda) - 3e^{2\phi}\partial\lambda\partial^2\lambda\}$$
$$\delta\lambda = \epsilon^{\beta\alpha}\{\partial(e^{2\phi}\lambda(\partial^3\varphi - (\partial\varphi)^3)) + e^{2\phi}\partial\lambda(\partial^3\varphi - (\partial\varphi)^3) + \frac{3}{2}\partial^2(e^{2\phi}\lambda\partial^2\varphi)$$
$$-\frac{3}{2}e^{2\phi}\partial^2\lambda\partial^2\varphi - 3\partial^2(e^{2\phi}\partial\lambda\partial\varphi) - 3\partial(e^{2\phi}\partial^2\lambda\partial\varphi)\}$$
$$\delta(\partial\phi) = 2\epsilon^{\beta\alpha}e^{2\phi}F_2(\varphi,\lambda)$$
$$\delta\chi = \delta b = \delta c = \delta X^n = \delta\psi^n = 0 \tag{38}$$

As before, $\epsilon^{\beta-}, \epsilon^{\beta n}, \epsilon^{\beta\alpha}$ are the transformation parameters related to $L^{\beta-}, L^{\beta n}$ and $L^{\beta\alpha}$. It is now straightforward to show that the RNS action (1) is invariant under the set of the α-transformations (36) - (38). The proof is identical to the case of $L^{\beta+}$, demonstrated above. Similarly to the case of (33), (34) the variation of the matter part of the action (1) under the α-transformations (36) - (38) is cancelled by that of the superconformal ghost part.

To summarize, we have shown that the RNS superstring action in d dimensions is invariant under the set of $d+3$ nonlinear space-time transformations induced by the generators of $H_2 \sim H_{-4}$, in addition to $d+2$ α-symmetries of $H_1 \sim H_{-3}$ and $\frac{1}{2}(d+1)(d+2)$ Poincare symmetries, isomorphic to $SO(d,2)$ space-time isometries of AdS_{d+1}. Thus the total space-time symmetry group is upgraded to $SO(d+2,2)$, with one extra dimension contributed by generators of the ghost cohomology $H_1 \sim H_{-3}$ and another by $H_2 \sim H_{-4}$. Note that, while the symmetries induced by $H_1 \sim H_{-3}$ can be associated with those of $2T$-physics, there is no obvious $2T$ counterpart of the higher order symmetries of $H_2 \sim H_{-4}$. This concludes the proof that the generators of $H_2 \sim H_{-4}$ effectively induce another new hidden space-time dimension, in addition to the one produced by $H_1 \sim H_{-3}$, in agreement with the concept of " a ghost cohomology = a hidden space-time dimension", implying that each ghost hohomology $H_n \sim H_{-n-2}, n = 1,2,3,...$ induces a set of space-time symmetries descending from the extra dimension associated with H_n.

In the following section we will discuss the generalization of our results to the general case of $n \geq 3$.

3. $H_3 \sim H_{-5}$ and the Higher Cohomologies

The construction of the α-symmetry generators described above can be generalized to the case of higher ghost cohmologies as well, though manifest expressions for the generators

become more complicated. As before, it is useful to take the operators $F_1(X,\psi)$ and $F_2(\varphi,\lambda)$ (26), (32) (structurally related to the matter part of Cartan generators of $SU(3)$ and $SU(4)$ in the $c=1$ model) as the building blocks to construct the currents of $H_3 \sim H_{-5}$. Since F_1 and F_2 are primary fields of dimensions $\frac{5}{2}$ and 5 respectively, one has $T(z)\partial F_1(X,\psi)(w) \sim \frac{5F_1(X,\psi)(w)}{(z-w)^3}$ and $T(z)\partial F_2(\varphi,\lambda)(w) \sim \frac{10F_2(w)}{(z-w)^3}$ and therefore

$$R = 2\partial F_1(X,\psi)F_2(\varphi,\lambda) - F_1(X,\psi)\partial F_2(\varphi,\lambda)$$

is a primary field of dimension $\frac{17}{2}$. Also $\Gamma(z) : F_1(X,\psi) : (w) \sim O((z-w)^{-2})$ and $\Gamma(z) : F_2(\varphi,\lambda) : (w) \sim O((z-w)^{-3})$ since $e^{-3\phi}F_1$ and $e^{-4\phi}F_2$ are the elements of H_{-3} and H_{-4} respectively. Therefore, as the field contents of F_1 and F_2 do not mix (one depends on X and ψ and another on the super Liouville mode), $\Gamma(z)R(w) \sim O((z-w)^{-4})$. Since $e^{-5\phi}$ has conformal dimension $\frac{15}{2}$ and $\Gamma(z)e^{-5\phi} \sim O((z-w)^5)$ (disregarding the irrelevant $c\partial\xi$ term of Γ), the operator $\oint \frac{dz}{2i\pi}e^{-5\phi}R$ is the integral of primary field of dimension 1 annihilated by picture-changing, i.e. the element of H_{-5}. This allows us to identify the first generator of $H_{-5} \sim H_3$. Introducing the space-time index γ associated with $H_{-5} \sim H_3$ (in addition to α and β) we identify

$$L^{\gamma+} \equiv T = \oint \frac{dz}{2i\pi}e^{-5\phi}(2\partial F_1(X,\psi)F_2(\varphi,\lambda) - F_1(X,\psi)\partial F_2(\varphi,\lambda)) \tag{39}$$

Using the same procedure as in the $H_{-4} \sim H_2$ case, it is not difficult to construct the remaining $d+3$ currents generating the space-time α-symmetries on the level of $H_{-5} \sim H_3$:

$$\begin{aligned} L^{\gamma m} &= [L^{\gamma+}, L^{-m}]; m = 0, ..., d-1 \\ \eta^{mn}L^{\gamma-} &= [L^{\gamma m}, L^{-n}] \\ L^{\gamma\alpha} &=:: \Gamma^3 : [L^{\gamma-}, L^{\alpha+}] : \\ L^{\gamma\beta} &=:: \Gamma^4[L^{\gamma-}, L^{\beta+}] :\equiv \oint \frac{dz}{2i\pi}e^{-5\phi}F_3(\varphi,\lambda) \end{aligned} \tag{40}$$

where the picture-changings in the last two commutators are necessary to bring them back to the original picture -5. As in the case of $H_{-4} \sim H_2$, the scalar generator $L^{\gamma\beta}$ mixing the Liouville mode and the ghosts (but commuting with X and ψ) coincides with the one of the Cartan generators of $c=1$ model - this time, with the $H_{-5} \sim H_3$ generator of $SU(5)$, with F_3 being its matter part. As before, this generator can be obtained by the prescription described in [6], namely, by applying the lowering $SU(2)$ currents 4 times to $\oint \frac{dz}{2i\pi}e^{-5\phi+4iX}\psi$:

$$\oint e^{-5\phi}F_3(\varphi,\lambda) \equiv: (\oint e^{-i\varphi}\lambda)^4 \oint e^{-5\phi+4i\varphi}\lambda : \tag{41}$$

Alternatively, instead of starting with $L^{\gamma +}$ (which form we simply have guessed) one could proceed in a more systematic way, deducing $L^{\gamma\beta}$ from the $H_{-5} \sim H_3$ Cartan generator of SU(5) in $c = 1$ model, replacing the $c = 1$ matter fields with φ and λ and then constructing other currents as the commutators of $L^{\gamma\beta}$ with the elements of the previous cohomologies. However, the latter procedure is more complicated from the technical point of view, as it involves more unpleasant picture changing transformations and in addition the expressions for the $SU(n)$ Cartan generators from cohomologies of high ghost numbers are quite lengthy. As before, $d+4$ generators (39),(40) of $H_{-5} \sim H_3$ combined with those of $H_{-4} \sim H_2$,$H_1 \sim H_{-3}$ and the Poincare generators of $SO(d,2)$, constitute $\frac{(d+3)(d+4)}{2}$ generators of $SO(d+3,2)$. These $d+4$ generators induce another set of higher order α-symmetry transformations at the level of $H_{-5} \sim H_3$, originating from another hidden space-time dimension, labelled with the index γ. By induction, this construction can be generalized to the cohomologies $H_n \sim H_{-n-2}$ of arbitrary n as well. In case of $n \geq 4$, however, there seems to be no easy way of deducing the form of the matter+Liouville generators of $H_n \sim H_{-n-2}$ from the Cartan building blocks, as we did above with $L^{\alpha+}$,$L^{\beta+}$ and $L^{\gamma+}$. So we have to start from the $H_n \sim H_{-n-2}$ Cartan generators of $SU(n+2)$. Introducing the index α_n for the extra dimension number n associated with the generators of $H_n \sim H_{-n-2}$, we identify the Cartan generator with

$$L^{\alpha_n\alpha_{n-1}} =: (\int \frac{dz}{2i\pi} e^{-i\varphi}\lambda)^{n+1} \oint \frac{dz}{2i\pi} e^{-(n+2)\phi+i(n+1)\varphi}\lambda : \tag{42}$$

(again, we consider the special case of zero dilaton field so there is no Liouville dressing). As before, apart from $L^{\alpha_n\alpha_{n-1}}$ there are n additional scalar generators of $H_n \sim H_{-n-2}$, constructed as

$$\begin{aligned} L^{\alpha_n\alpha_i} &= [L^{\alpha_n\alpha_{n-1}}, L^{\alpha_{n-1}\alpha_i}], i = 1, ..., n-2 \\ L^{\alpha_n\pm} &= [L^{\alpha_n\alpha_{n-1}}, L^{\alpha_{n-1}\pm}] \end{aligned} \tag{43}$$

and one d-vector generator:

$$L^{\alpha_n m} = [L^{\alpha_n\alpha_{n-1}}, L^{\alpha_{n-1}m}] \tag{44}$$

so altogether there are $d+n+1$ generators in the $H_n \sim H_{-n-2}$ ghost cohomology, generating the α-symmetry transformations associated with the nth hidden space-time dimension. Combined with the generators of the lower cohomologies $H_k \sim H_{-k-2}$ with $0 \leq k \leq n-1$

these currents would generate the $SO(d+n,2)$ space-time symmetry group. Unfortunately, because of the complexity of the manifest expressions for the currents of the higher cohomologies, we have not been able to verify this explicitly for $n \geq 4$.

4. Conclusions

The main result of this work shows that RNS superstring theories in various dimensions possess a hierarchy of space-time symmetries (α-symmetries), realized nonlinearly. Eash class of α-symmetries is generated by the currents associated with ghost cohomology $H_n \sim H_{-n-2}$ of number n . By explicit construction, we have shown that each ghost cohomology $H_n \sim H_{-n-2}$ has $d+n+1$ elements, namely, $n+1$ scalar generators and one $SO(d-1,1)$ vector, inducing $d+n+1$ space-time transformations. The RNS superstring action is invariant under these transformations with the variation of the matter part cancelled by that of the ghost part. Combined together, the generators from cohomologies $H_k \sim H_{-k-2}(0 \leq k \leq n)$ generate the $SO(d+n,2)$ space-time symmetry group which is larger than the "naive" SO(d,2) Poincare group of non-critical RNS superstring theory in d dimensions (i.e. the isometry group in the absence of the α-transformations mixing the matter and the ghost fields). Thus each class of the α-symmetries corresponding to $H_k \sim H_{-k-2}(1 \leq k \leq n)$ can be attributed to a hidden space-time dimension (so altogether n ghost cohomologies generate n extra dimensions). In the simplest case of $n=1$, the α-symmetry generators on the level of $H_1 \sim H_{-3}$ are in one to one correspondence to the generators of the nonlinear space-time symmetries for a AdS_d particle, observed in the $2T$-formalism and which are also linked to a hidden extra dimension in the Bars approach [3], [4], [5]. Our approach suggests, however, that the list of symmetries observed in the $2T$ physics for a particle, is not complete as it corresponds only to the lowest level α-transformations of $H_1 \sim H_{-3}$. Therefore an interesting question is whether there is any interpretation of the higher level α-symmetries in the language of $2T$ physics. Operators from higher ghost cohomologies , discussed in this paper, suggest that the number of hidden dimensions is bigger than 1, therefore the analogue of higher level α-symmetries should also exist for point particles, though these extra symmetries have not yet been detected in the $2T$ approach. In this work we have studied the hierarchy of α-symmetries on the classical level, i.e. as the space-time symmetry transformations of the worldsheet RNS action. The important step forward would be to generalize the discussion to the quantum level, in particular, to point out the behaviour of the S-matrices and the correlators under the α-transformations and the related conservation laws. In other words, what are the charges conserved as a result of the α-symmetries and what is their physical significance?

For example, in the simplest case of the $c = 1$ model the generators of $H_1 \sim H_{-3}$, enhancing the current algebra from the standard $SU(2)$ to $SU(3)$, imply that the tachyonic highest weight vectors and their descendants at discrete momenta possess a new quantum number interpreted as "hypercharge", conserved in interactions of the ghost-dependent discrete states [6]. While the $c = 1$ model is mainly an elegant toy to play with (rather than a realistic phenomenological model), so the "hypercharge" of discrete vertex operators is hardly of any phenomenological significance, the appearance of new non-trivial conservation laws related to the interactions of ghost-dependent discrete states, is by itself remarkable. One can ask if the conservation laws associated with the α-symmetries in higher dimensional RNS superstring have any phenomenological interpretation. It is possible that such an interpretation could lead to some interesting stringy scenarios of strong or even electroweak interactions.

In our paper we have investigated the limit of zero dilaton field, in order to avoid complications related to the Liouville background charge. In principle, it is straightforward to generalize our discussion to include the effects of the dressing, though manifest expressions for the generators become more entangled, except for the case of $d = 9$ when the background charge is absent. (alternatively, one can consider a critical string theory in $d = 10$ and compactify one of the dimensions on S^1). The $d = 9$ or the compactified $d = 10$ cases are of the special interest since the generators of $H_1 \sim H_{-3}$ bring us immediately to $d = 11$, relevant to the M-theory dynamics while " switching on" the higher cohomologies such as $H_2 \sim H_{-4}$ and $H_3 \sim H_{-5}$ would further advance us to the framework of F and S-theories (compactified on a circle, if one chooses to start from critical strings on S^1) [10], [11]. The problem is how the appearance of the higher dimensions can be explained dynamically. To address this question, one has to investigate the worldsheet renormalization group flows, induced by the vertices of nonzero cohomologies in the RNS sigma-model. These RG flows are known to be stochastic, described by the Langevin-type equations with the stochastic time, given by the log of the worldsheet cutoff, that also plays the role of the extra dimension [6]. So far these RG flows have only been explored in the simplest case of $H_1 \sim H_{-3}$ operators with only one stochastic time and one extra dimension present. As each cohomology contributes its own associate hidden dimension, switching on the operators from higher H_n's naturally directs us to the concept of the stochastic processes with multiple fictitious time variables. It seems that no systematic understanding of such processes exists at present, and this by itself is of some interest. Finally, another question for the future research is related to the special case of critical RNS

superstrings compactified on a circle of self-dual radius. The case of the self-dual radius is peculiar because the number of space-time symmetry generators is larger: firstly, one still can build the α-symmetry part of $SO(d+n,2)$ on the basis of $SU(n+2)$ Cartan generators of supersymmetric $c=1$-model , as has been shown in this paper. These currents all carry momentum zero. On the other hand, there are also the generators with discrete momenta in the compactified direction, inheriting their structure from all the $SU(n+2)$ generators of the $c=1$ model, in addition to those rooted in the Cartan subalgebra. This enhanced space-time symmetry, appearing at the self-dual compactification radius, needs a separate investigation and may involve interesting relations between S and T dualities, as well as phenomenological implications.

Appendix

Here we present the details of the calculation showing the absense of the Liouville-independent space-time vectors in $H_2 \sim H_{-4}$. The most general expression for the vector generator of dimension 1 in the picture -4 is given by

$$
\begin{aligned}
V^n = &\alpha_1(\partial X_m\partial X^m)(\psi_l\partial\psi^l)\partial X^n + \alpha_2(\psi_m\partial X^m)(\partial\psi_l\partial X^m)\partial X^n \\
&+\alpha_3(\psi_m\partial X^m)(\psi_l\partial^2 X^l)\partial X^n + \alpha_4(\partial X_m\partial X^m)^2\partial X^m \\
&+\beta_1(\partial X_m\partial X^m)(\psi_l\partial\psi^l)\partial\psi^n + \beta_2(\partial X_m\partial^2 X^m)\psi^n + \beta_3(\partial X_m\partial X^m)(\partial\psi_l\partial X^l)\psi^n \\
&+\beta_4(\partial X_m\partial X^m)(\psi_l\partial^2 X^l)\psi^n + \gamma_1(\psi_m\partial\psi^m)(\psi_l\partial X^l)\partial\psi^n + \gamma_2(\psi_m\partial^2\psi^m)(\psi_l\partial X^l)\psi^n \\
&+\gamma_3(\psi_m\partial\psi^m)(\partial\psi_l\partial X^l)\psi^n + \gamma_4(\psi_m\partial\psi^m)(\psi_l\partial^2 X^l)\psi^n + \lambda_1(\psi_m\partial^3\psi^m)\partial X^n \\
&+\lambda_2(\partial\psi_m\partial^2\psi^m)\partial X^n + \lambda_3(\psi_m\partial^2\psi^m)\partial^2 X^n + \lambda_4(\psi_m\partial\psi^m)\partial^3 X^n \\
&+\rho_1(\psi_m\partial X^m)\partial^3\psi^n + \rho_2(\psi_m\partial^2 X^m)\partial^2\psi^n + \rho_3(\partial\psi_m\partial X^m)\partial^2\psi^m + \rho_4(\partial^2\psi^m\partial X^m)\partial\psi^n \\
&+\rho_5(\partial\psi_m\partial^2 X^m)\partial\psi^n + \rho_6(\psi_m\partial^3 X^m)\partial\psi^n + \rho_7(\psi_m\partial^4 X^m)\psi^n + \rho_8(\partial\psi_m\partial^3 X^m)\psi^n + \rho_9 \\
&(\partial^2\psi_m\partial^2 X^m)\psi^n + \rho_1 0(\partial^3\psi_m\partial X^m)\psi^n + \sigma_1\partial^5 X^n + \sigma_2(\partial X_m\partial^3 X^m)\partial X^n + \\
&\sigma_3(\partial^2 X_m\partial^2 X^m)\partial X^n + \sigma_4(\partial X_m\partial^2 X^m)\partial^2 X^n + \sigma_5(\partial X_m\partial X^m)\partial^3 X^n
\end{aligned}
\tag{45}
$$

where $\alpha,\beta,\gamma,\lambda,\rho$ and σ are some coefficients. Computing OPE of V^n with the stress tensor gives 22 primary field constraints on V^n following from the condition that the OPE coefficients in front of all of the operators appearing in terms of the order of $(z-w)^{-n}(n\geq 3)$ must vanish, i.e. the OPE has no singularities higher than quadratic. That is, in our case, the most singular OPE term (for generic $\alpha,\beta,\gamma,\lambda,\sigma,\rho$'s and δ_1) is of the order of $n=-6$. Subsequently, the constraints for vanishing of singularities of the order $n=-4,-5,-6$ give the first 9 linear equations of 22:

$$
\begin{aligned}
(12d+2)\sigma_2+8d\sigma_3+8\sigma_4+24\sigma_5-18d\lambda_1-2d\lambda_2-18\rho_1-2\rho_3+2\rho_4+18\rho_{10}-240\delta_1&=0\\
8\sigma_3+(4d+4)\sigma_4-4d\lambda_3-4\rho_2+4\rho_9-240\delta_1&=0\\
(4d+16)\alpha_4-d\alpha_1-\alpha_2-\beta_1+\beta_3-12\sigma_5-12\sigma_2&=0\\
4\alpha_3+(4d+4)\beta_2+8\beta_4+(8-4d)\gamma_2-48\rho_7-24\rho_{10}-24\rho_1&=0\\
(2d+4)\alpha_1+2\alpha_2-\gamma_1+\gamma_3-12\lambda_4-30\lambda_1+6\lambda_2&=0\\
2\alpha_2+(2d+4)\beta_1-d\gamma_1-12\rho_6-12\lambda_4-30\rho_1-6\rho_4&=0\\
(2d+4)\beta_3-d\gamma_3-12\rho_8-30\rho_{10}-6\rho_3&=0\\
2\alpha_2+2\beta_2+2d\beta_4+(2-d)\gamma_4-6\rho_2-48\rho_7-6\rho_9&=0\\
2\sigma_2+2d\sigma_5-d\lambda_4-\rho_6+\rho_8-120\delta_1&=0
\end{aligned}
\tag{46}
$$

The vanishing of the cubic terms gives the remaining 13 primary field constraints:

$$
\begin{aligned}
4\sigma_4+12\sigma_5=0;4\sigma_4+12\sigma_2+8\sigma_3=0;4\rho_9+18\rho_{10}+2\rho_4&=0\\
2\rho_5+8\rho_9+12\rho_8=0;2\rho_6+24\rho_7+2\rho_8=0;8\rho_3+8\rho_4+4\rho_5&=0\\
8\rho_2+2\rho_5+12\rho_6=0;18\rho_1+4\rho_2+2\rho_3=0;18\lambda_1+2\lambda_2+4\lambda_3&=0\\
9\lambda_3+12\lambda_4=\beta_4+\beta_2=\delta_1&=0\\
2\beta_1+2\beta_3+2\gamma_1+6\gamma_2+2\gamma_3+4\gamma_4+2\lambda_2&=0
\end{aligned}
\tag{47}
$$

The next set of constraints are the annihilation conditions by Γ, $\Gamma V\sim 0$. The analogous calculation gives further 5 constraints on the coefficients of (45), following from the vanishing of all the OPE terms of the orders of $(z-w)^{-n}(n=0,1,2)$ to ensure the annihilation by the direct picture changing:

$$
\begin{aligned}
3\lambda_1+2\lambda_3+3\lambda_4+3\rho_1+2\rho_2+3\rho_6+12(1-d)\rho_7-3d\rho_8-2d\rho_9-3d\rho_{10}&=0\\
\lambda_2-3\lambda_4+\rho_3-d\rho_4+(1-d)\rho_5-3d\rho_6+3\rho_8&=0\\
2\alpha_1+(d+2)\alpha_2+2(d-1)\alpha_3+2\beta_1+2\beta_2-6\lambda_1-6\rho_1+6\sigma_2&=0\\
\alpha_1+\beta_1-2\beta_2-(d+2)\beta_3+2(d-1)\beta_4+6\sigma_5+6\rho_{10}&=0\\
\gamma_1+(1-d)\gamma_3+(4-2d)\gamma_4+6\lambda_4+6\rho_8&=0
\end{aligned}
\tag{48}
$$

The constraints (46) - (48) are the necessary, but not the sufficient cohomology conditions for V. In order to be the cohomology element generating global space-time symmetries, V also has to be BRST non-trivial. Conversely, to verify the BRST non-triviality, it

is sufficient to require that the action (1) is invariant under the space-time transformations induced by the worldsheet integral of the operator (45). The straightforward calculation of the variations under the space-time transformations induced by (45), along with some partial integration, shows that the invariance of the action leads to the set of 29 further constraints on the coefficients of (45):

$$
\begin{aligned}
\rho_1 - \rho_2 + \rho_6 - 4\rho_7 + \rho_8 - \rho_9 + \rho_{10} &= 0 \\
3\rho_1 - 2\rho_2 - \rho_3 + \rho_5 + \rho_6 - 2\rho_8 - 2\rho_9 &= 0 \\
3\rho_1 - \rho_2 - 2\rho_3 + \rho_4 + \rho_5 - 4\rho_9 &= 0 \\
\rho_1 - \rho_3 + \rho_4 - \rho_7 + \rho_8 - \rho_9 - 2\rho_{10} &= 0 \\
-\rho_4 + \rho_5 - 3\rho_6 + \rho_8 - 2\rho_9 + 3\rho_{10} = 0; -\rho_4 - \rho_5 + \rho_6 - 3\rho_7 + 2\rho_8 - \rho_9 &= 0 \\
-\rho_2 + \rho_3 - 2\rho_4 + \rho_5 - 3\rho_9 + 3\rho_{10} = 0; -\rho_2 - \rho_3 - \rho_5 + 2\rho_6 - 3\rho_7 + \rho_8 &= 0 \\
-2\rho_1 - \rho_2 + \rho_3 - \rho_4 + \rho_6 - \rho_7 + \rho_{10} &= 0 \\
\rho_5 = 2\lambda_2 - 3\lambda_1 = -3\lambda_2 - \lambda_3 + \lambda_4 = -3\lambda_1 + 3\lambda_2 - 2\lambda_3 &= 0 \\
-3\lambda_1 + \lambda_2 + 2\lambda_3 - 4\lambda_4 = 0; \sigma_2 - 2\sigma_3 - \sigma_4 + 2\sigma_5 = 0; -\sigma_3 - \sigma_4 + 2\sigma_5 &= 0 \\
\sigma_2 - \sigma_3 - \sigma_4 = 0; \sigma_2 - \sigma_4 &= 0 \\
2\beta_1 - 3\beta_2 + 2\beta_3 - 2\beta_4 = 0; \beta_1 - \beta_2 + \beta_3 - 2\beta_4 = 0; \beta_1 - \beta_2 - \beta_4 &= 0 \\
\beta_3 - \beta_2 - \beta_4 = 0; \alpha_1 = \alpha_2 = \alpha_3 = 0; \gamma_1 - 3\gamma_2 + \gamma_3 - \gamma_4 &= 0 \\
\gamma_1 + 2\gamma_2 + \gamma_3 - 6\gamma_4 = 0; \gamma_1 + 2\gamma_2 - 4\gamma_3 - \gamma_4 = 0; -4\gamma_1 + 2\gamma_2 + 2\gamma_3 &= 0
\end{aligned}
\tag{49}
$$

This is the set of 29 constraints for 31 coefficients so the system has at least 2 independent nonzero solutions (actually the number of independent solutions is larger since the system is degenerate). Any choice of coefficients in (45) satisfying (49) gives a space-time symmetry generator, so the number of the symmetry generators is equal to the number of linearly independent solutions of (49). Now there are two possibilities: the first is that all these generators differ only by BRST-trivial terms and are related to the usual translation generator, transformed to picture +2. This is the case if the solutions aren't compatible with the annihilation constraints (48) (note that the picture +2 translation is by construction a primary field, so (46) and (47) are satisfied automatically). The second possibility is that the solutions of (49) satisfy (46), (47) and (48). In this case the generator is the element of ghost cohomology and induces the α-symmetry transformations on the level of $H_2 \sim H_{-4}$. Though we have described the case of generators at pictures -4 or $+2$, the same logic of

search for α-symmetries applies to case of operators at higher ghost numbers as well, for this reason we felt it would be instructive to demonstrate the above calculations in this appendix.

Since the overall number of constraints (46) - (49) is bigger than the number of the coefficients in (45), the appearance of the α-symmetry is possible only in case of the degeneracy of the linear system of equations induced by the primary, annihilation and symmetry constraints altogether.

It is not difficult to check that in the case under consideration (picture -4 or $+2$ vector generators) the constraints (46) - (49) have no nonzero solutions, therefore there are no Liouville-independent generators of the α-symmetry at this level. The same can be shown for higher rank tensors as well. This result ensures that we have no excessive α-generators and the set of the currents (27) and (30) - (32), inducing the α-symmetries at the level $H_2 \sim H_{-4}$, is complete.

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